

## Integrated nanolasers via complex engineering of radiationless states

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PERSPECTIVE • **OPEN ACCESS**

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## PERSPECTIVE

## Integrated nanolasers via complex engineering of radiationless states

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## Abstract

The development of compact and energy-efficient miniaturised lasers is a critical challenge in integrated non-linear photonics. Current research focuses on the integration of subwavelength all-dielectric lasers in CMOS compatible platforms. These systems provide a viable alternative to state-of-the-art nanoplasmonic sources, whose practicality is often hindered by high metal losses. The efficiency of dielectric nanolasers, however, is affected by the diffraction limit of light, which restricts the degree of localisation achievable with standard resonator modes. The recent development of new types of radiationless states has brought a sharp innovation in the field of subwavelength dielectric lasers. Radiationless states are exotic electromagnetic solutions that originate from the complex superposition and interaction of several resonator modes. They are associated with a high degree of near-field localisation which makes them particularly advantageous for non-linear photonics applications. In this work, we provide an overview of the most recent theoretical and experimental efforts toward the development of integrated lasers and ultrafast sources based on the amplification of exotic radiationless states. In particular, we focus our attention on two specific types of radiationless states: optical anapoles and Bound States in the Continuum (BIC). By discussing their differences and similarities, we provide a unifying view of these distinct research areas and outline possible future directions for these innovative platforms.

## 1. Introduction

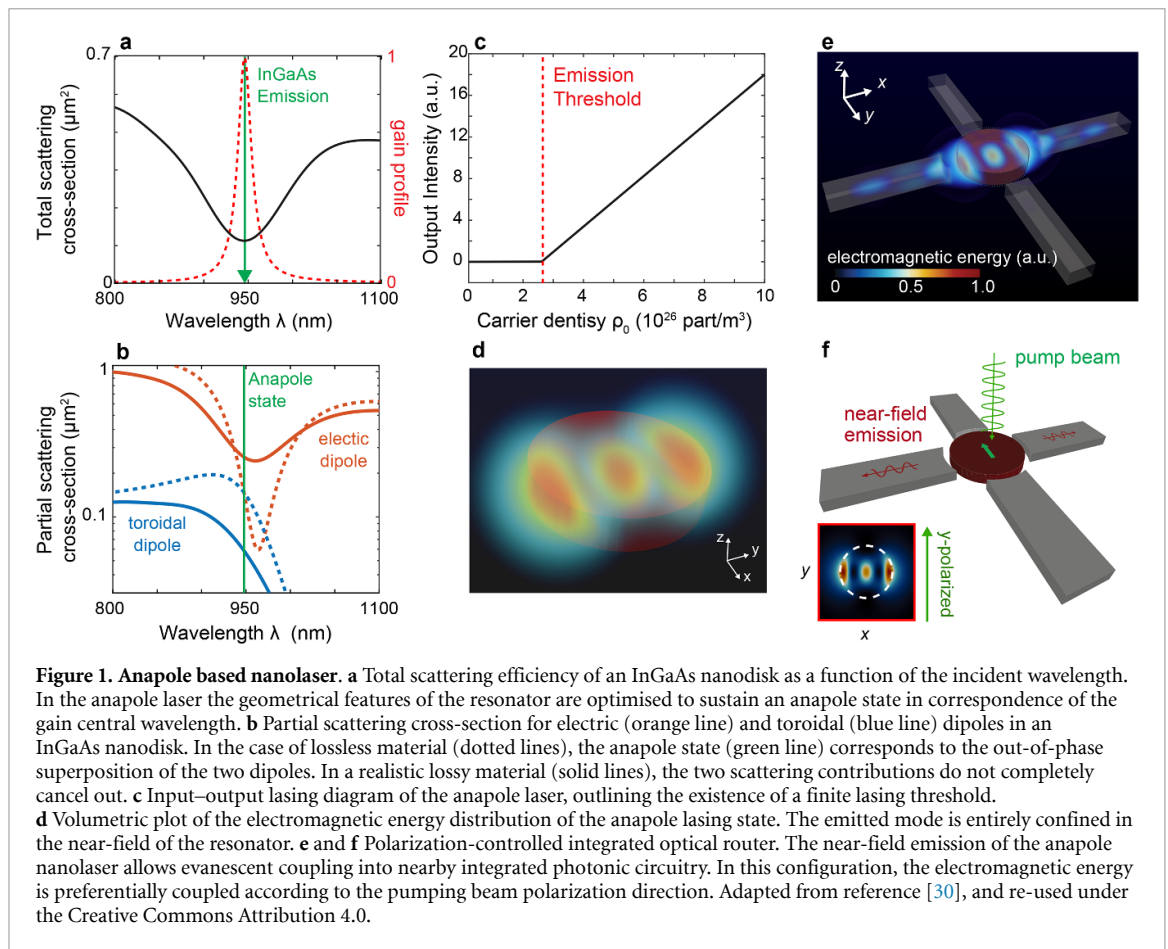
The miniaturisation of ultrafast lasers in integrated photonic devices is a key challenge in non-linear photonics [1]. The development of an energy-efficient on-chip source could enable new generations of devices in critical application areas, including portable atomic clocks, wearable medical devices, mobile telecommunications, and optical computing [2–4]. Current research aims to tackle the intrinsic trade-off between the energy efficiency of an on-chip laser, the compatibility with established CMOS technology, and achieving the highest degree of miniaturisation [1, 5]. To date, state-of-the-art on-chip lasers rely mostly on extended systems with footprints of several  $\mu\text{m}^2$  (e.g. VCSELs, photonic crystals or micro-ring resonators) [5, 6]. While most applications would prefer more compact footprints, there are increasing practical and conceptual challenges when trying to push the on-chip laser footprint below the wavelength scale. Most of these limitations stem from the fundamental electromagnetic properties of subwavelength resonators. At these scales, the diffraction limit of light hinders the possibility of achieving lasing emission with low pumping powers by limiting the ability to localise and enhance the electromagnetic fields within the gain region [7]. In subwavelength resonators, low-order radiation modes with significantly small Q-factors dominate the system response [8]. These radiation modes, known as bright modes or leaky resonances, couple quite efficiently to far-field radiating waves. As these states offer only a limited amount of field localization, they are mostly unsuitable for applications requiring strong field confinement, such as in the case of laser systems. Under specific conditions, nanophotonic structures can support a different type of

resonator modes, denoted as dark modes, which are characterised by minimal coupling with radiating modes, exponential field attenuation, and remarkably high Q-factors [9]. Dark modes are ubiquitous in nano-metallic systems, where they are associated with the excitation of localized Surface Plasmon Polaritons (SPPs) [10]. The exploitation of the significant field enhancement of dark states to reduce the lasing threshold of miniaturised sources and boost their efficiency has been the subject of intense research [11–13]. One of the most promising candidates are plasmonic nanolasers, or SPASERs, which rely on the amplification of localized plasmonic modes [14–16]. These devices, however, are generally characterised by prohibitive material losses and high lasing thresholds which limit their practicality in real-life, integrated photonic devices [7, 17]. As an alternative, recent research focused on the development of integrated sources based on all-dielectric material platforms. In these systems, however, the excitation of dark states is a significantly challenging task. First of all, dark modes are mostly inexistent in subwavelength dielectric structures. Secondly, and most importantly, the purely evanescent nature of dark modes makes them significantly difficult to excite through far-field optical illumination [18]. Recent developments in the design of high-confinement subwavelength modes in subwavelength resonators could provide a game-changing solution to this challenge [19, 20]. Several research groups have demonstrated how to engineer non-trivial electromagnetic responses by taking advantage of the vibrant landscape of resonant modes in subwavelength dielectric resonators [21, 22]. These results suggest that it is possible to engineer a special class of radiationless super-modes through the superposition and interaction of separate leaky resonances [23]. These radiationless states possess the character of quasi-dark states, as they allow high field confinement by suppressing coupling to external radiation channels [20]. In this work, we discuss two specific types of quasi-dark cavity supermodes which are particularly relevant to the development of integrated subwavelength lasers: anapole states and Friedrich-Wintgen Bound states In the Continuum (FW-BIC). Similar to standard dark modes, these exotic states provide a significant near-field enhancement which ensures lower lasing thresholds when compared with standard bright resonances [20]. At the same time, these radiationless modes inherit from their modal composition the ability to be efficiently excited with far-field illumination. Both these features are uniquely advantageous to the development of efficient and tuneable ultrafast integrated lasers based on CMOS-compatible material platforms. Quite interestingly, the emission from these sources is localised in the near-field, opening to the possibility of realising optical interconnects and biological sensors which are currently out of reach with standard nanolasers radiating to free space [3].

## 2. The anapole-based nanolaser: stimulated emission and synchronization of near-field non-resonant states

Anapole states are exotic radiationless states characterised by a sharp reduction of the scattering from the dielectric nanoparticle [24]. These states originate from the non-resonant superposition of different dipole modes which cancel each other in the far-field and produce a strongly confined radiation pattern in the near-field [25]. A series of recent theoretical and experimental results have demonstrated that the near-field enhancement associated with anapole states can greatly enhance non-linear processes either in proximity or within the dielectric resonator. In the case of germanium nanodisks, for example, the excitation of fundamental and high-order anapole states provides a significant enhancement of non-linear frequency conversion processes [26–28]. Anapole states in Si nanoparticles can be also coupled with metallic structures or slots to reinforce the near-field enhancement and boost non-linear phenomena [29]. In the context of integrated lasers, the concept of a near-field source emitting at the anapole wavelength was first proposed in [30]. In this work, the nanolaser consisted of an optically-pumped InGaAs nanodisk of diameter  $d = 560$  nm emitting at a wavelength  $\lambda = 948$  nm. The authors optimised the resonator geometry to support an anapole mode, identified by a dip in the total scattering cross-section  $C_{scat}$ , in correspondence of the semiconductor emission wavelength that is determined by the Indium concentration in the III-V semiconductor (figure 1(a)). As highlighted by the multipole decomposition of the scattered field (figure 1(b), solid lines), the presence of material losses in the gain medium leads to a partial decoupling of the electric and toroidal dipoles, which would precisely cancel out in the lossless case (figure 1(b), dashed lines). Despite the extremely reduced footprint, the authors demonstrated through *ab-initio* electromagnetic simulations equations demonstrated that the anapole-based nanolasers possess the typical emission properties of an integrated laser, as illustrated by the input-output lasing diagram in figure 1(c). Differently from a standard nanoparticle laser [31], however, in this case, the emission is entirely confined in the near-field (figure 1(d)).

The tightly localized near-field profile of the anapole emission opens to the possibility of coupling efficiently with passive nanophotonic circuitry. In [30], the authors demonstrated not only that the anapole emission could couple into integrated silicon waveguides more efficiently than standard low-Q modes, but also that its mode profile allowed controlling the directionality of the coupling. This concept was



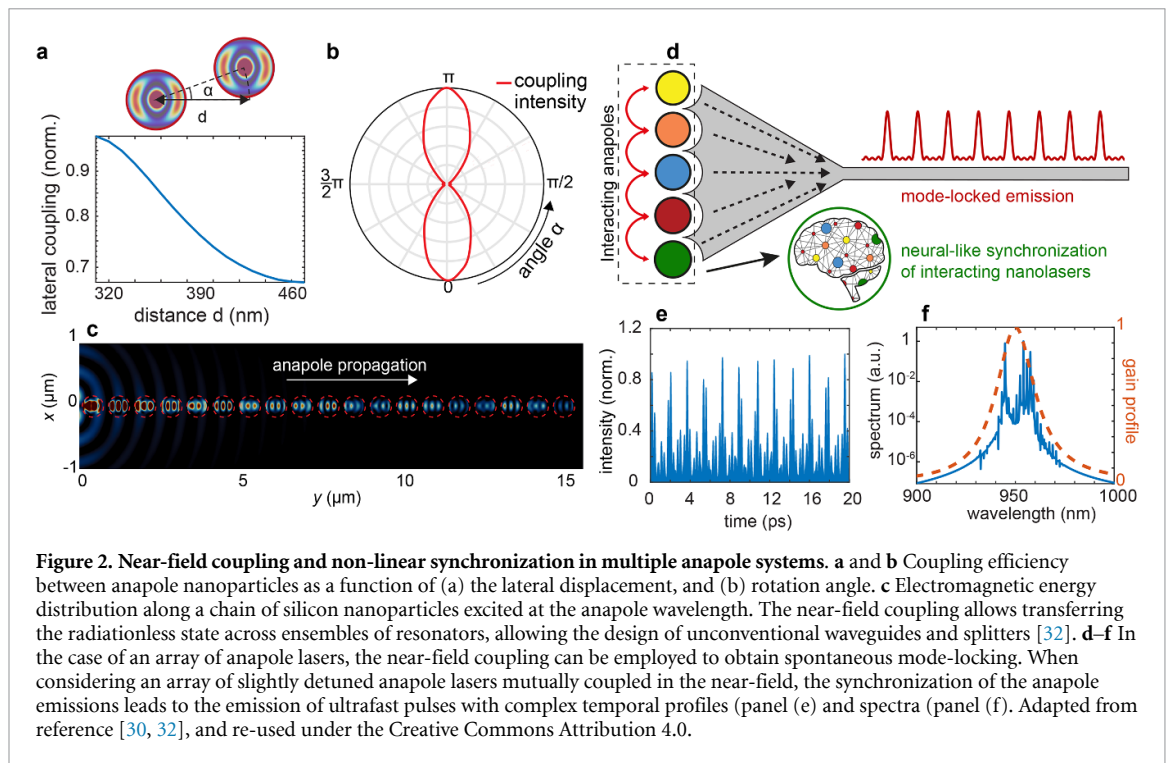
demonstrated in a polarization controlled integrated router entirely controlled through the direction of the optical pump polarisation (figures 1(e) and (f)).

An interesting question is whether the near-field coupling properties of the anapole can provide an edge to engineer advanced devices based on complex interactions between different anapole states. To answer this question, in [32] Mazzone *et al* investigated the mutual coupling of passive anapole nanoparticles. In figures 2(a)–(c) we report the results of their analysis on Si nanodisks, which confirmed how the mutual coupling of different anapole states is strongly dependent on both their position (figure 2(a)) and orientation (figure 2(b)), as initially suggested in [30]. As a direct application, the authors demonstrated that the non-radiating state could be efficiently transferred across chains of nanoparticles (figure 2(c)), and that the guiding properties of the anapole chains were robust against bends and splitting [32].

The possibility of engineering complex coupling and interactions across chains of anapole nanoparticles opens entirely new possibilities in the presence of non-linear gain media. In arrays of anapole-based nanolasers, for example, the near-field coupling can be exploited to precisely control the non-linear interaction between distinct anapole sources. This idea was explored in [30], where the authors coupled the collective emission from a chain of InGaAs anapole-based nanolasers into a silicon waveguide (figure 2(d)). The geometry of each anapole nanolaser was optimised to sustain lasing emission at a slightly different wavelength. Differently from the passive chain of [32], in this configuration, the presence of optical gain leads to complex synchronisation processes that are fertile ground for the realisation of advanced ultrafast sources. The relative displacement of the individual anapole lasers in the chain, in particular, can be optimised to achieve spontaneous mode-locking of the integrated lasers (figures 2(e) and (f)). In this condition, the emission from the anapole array can spontaneously lock through a long-range synchronisation mechanism analogous to the ones found in fireflies and biological neurons [33–35].

### 3. Further developments: thresholdless nanoparticle lasers based on radiationless BIC states

Even when considering these unique features and applications, the possibility of engineering integrated sources with anapole states appears quite counter-intuitive. Differently from dark modes and plasmons, anapole states constitute a non-resonant superposition of multipole components which appears unsuitable

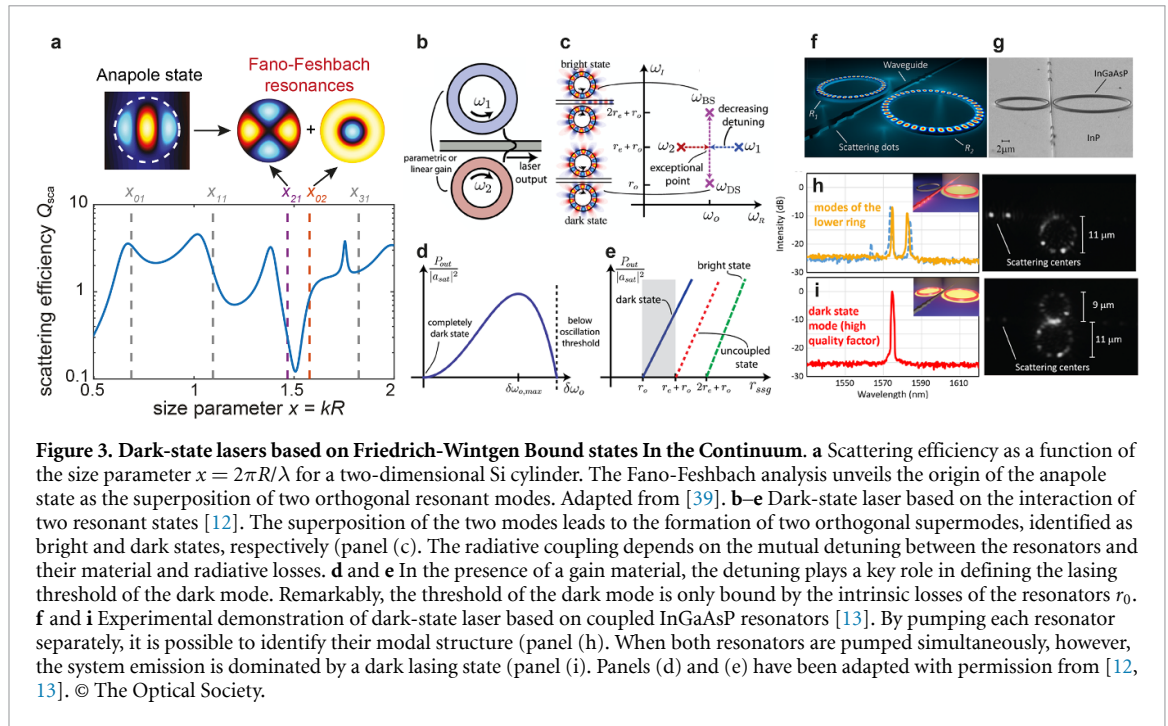


to sustain stimulated emission of radiation [24, 36]. This apparent contradiction, however, can be disentangled by taking a closer look at the fundamental properties of the anapole states. Historically, the onset of a photonic anapole was explained through complex multipole expansions [37, 38]. In this representation, the anapole corresponds to a specific combination of electric and toroidal dipoles [25]. At the anapole frequency, these modes interfere destructively in the far-field and constructively in the near-field, yielding the strong near-field enhancement highly sought for non-linear applications. The anapole state, as such, appears as a dynamic ‘equilibrium state’ of the electromagnetic fields and it is generally not associated with a finite Q-factor [19]. Such a dynamic state exists only under stationary illumination, and it disappears as soon as the external illumination is removed [36].

The description of this state with a more rigorous treatment has been developed in [39], where the authors re-defined the anapole state in terms of a complete set of orthogonal, resonant states. Since dielectric resonators are, by definition, open electromagnetic cavities, one cannot directly solve Maxwell’s equations by defining a set of orthogonal eigenmodes [40]. To overcome this limitation, the authors applied a Fano-Feshbach projection scheme to describe the electromagnetic scattering of a subwavelength resonator. This technique, which originated from the field of open quantum systems [41, 42], splits the resonator in two regions: an internal space comprising the resonator and supporting a discrete set of orthogonal modes, and an external space characterised by a continuum of radiative scattering eigenmodes. By coupling the two domains through *ad-hoc* boundary conditions, it is possible to describe the electromagnetic scattering from the resonator in terms of internal and external resonant eigenmodes. The authors demonstrated that the anapole state can be explained in terms of quasi-overlapping resonant states, known as Fano-Feshbach resonances (figure 3(a)). Also, they showed how the orthogonal expansion allows identifying higher-order anapole states that originate from the complex superposition of a large number of internal resonances of the structure. These states cannot be represented in terms of the fundamental anapole mode.

In these terms, the anapole states appear as the non-resonant counterpart of a specific type of cavity supermode known as Friedrich-Wintgen Bound states In the Continuum (FW-BIC) [19]. BIC states represent radiationless states which correspond to discrete resonances (bound states) localised in the radiation continuum spectrum of a photonic system [18]. Since BIC states are characterised by diverging Q-factors, these states are excellent candidates for low-threshold lasers. Generally speaking, the threshold of a nanolaser is defined by the minimum power required to achieve stimulated emission and overcome material and radiative losses [43]. As BIC states are characterised by vanishing radiative losses, their lasing threshold should be set only by the intrinsic material losses of the gain medium. To date, lasing action from BIC states has been demonstrated in extended photonic structures [44–46], but their extension to localised resonator structures is still a matter of intense research [20]. In this context, FW-BIC states are a particularly promising family of radiationless BIC states. Similarly to the anapole state, FW-BIC states originate from the





superposition of two different resonances in the same optical cavity [47]. Differently from the anapole, however, the BIC state is a resonant state, whose electromagnetic properties depend on the features of the individual resonances composing it. A detailed analysis of the differences between anapole and BIC states can be found in [19]. Due to their simplicity, FW-BIC states are among the best candidates for the realisation of low-threshold, subwavelength laser in all-dielectric platforms.

The concept of a tunable, low-threshold laser relying on an FW-BIC was initially proposed in [12]. In this work, Gentry and co-workers analysed a dark-state laser composed of two microring resonators with different resonant frequencies  $\omega_{1,2} = \omega_0 \pm \delta\omega_0$ . The two counter-propagating modes are mutually coupled by far-field interference in a shared radiation channel (figure 3(b)). In this configuration, the superposition of the two individual resonator modes produces two orthogonal supermodes (figure 3(c)) corresponding to a bright (symmetric) and a dark (antisymmetric) state. The Q-factors of the two supermodes can be adjusted by acting on the mutual detuning  $\delta\omega_0$ , and through the material  $r_0$  and radiative  $r_e$  losses of the two individual modes (figure 3(c)). To investigate the lasing characteristics of the dark-state laser, the authors introduced a saturable gain into the resonators. Their analysis shows that when the gain, described by the small-signal gain rate  $r_{sg}$ , lies in the interval  $r_0 < r_{sg} < r_e + r_0$ , i.e. it is contained between the material losses and the loaded passive decay rate, only the dark state will exceed the lasing threshold. The latter is ultimately bound by the material losses in the system. In the presence of identical cavities ( $\delta\omega_0 = 0$ ), the antisymmetric state is a perfect dark state which is uncoupled from the external channel. In the presence of a slight detuning, conversely, the radiative coupling of the dark state supermode into the radiating channel is expressed as  $r_{DS,e} = r_e - \sqrt{r_e^2 - \delta\omega_0^2}$ . In these conditions, the detuning controls both the lasing frequency of the dark-state and its coupling to the external waveguide, allowing to tune the threshold and peak emission from the miniaturised laser (figures 3(d) and (e)). Due to the simplicity of the micro-ring architecture, this type of dark state laser can be directly implemented using standard integrated photonics platforms. In [13], Hodaei and coworkers demonstrated a tunable dark state laser composed of two InGaAsP resonators/waveguide system (figures 3(f) and (g)). By individually exciting each resonator, the authors were able to selectively excite and observe lasing emission from the corresponding resonator modes (figure 3(h)). When both resonators were pumped simultaneously, they observed the excitation of the antisymmetric dark state. As predicted in [12], the dark state emission was characterised by a lower lasing threshold, higher slope efficiency, and enhanced total output power when compared with the standard modes from the individual resonators. Also, the lasing frequency was continuously tunable within an 8 nm hop-free range by acting on the mutual detuning through the ambient temperature.

An interesting question that is currently driving future research efforts is whether the advantages of a low-threshold FW-BIC laser could be integrated with the physics of subwavelength dielectric resonators. An ideal FW-BIC nanoparticle laser could exploit the superposition of two (or more) resonances to achieve a tunable, quasi-thresholdless near-field emission. In dielectric resonators, however, the open challenge is how


to individually control the wide number of overlapping resonances to ensure the formation of an FW-BIC state capable of supporting room-temperature, low-threshold lasing action. In this context, the emergence of quasi-BIC states in high-index dielectric nanoparticles constitutes, in our view, a very promising research direction [23, 48]. In these systems, it is possible to engineer the destructive interference between several leaky modes of a dielectric microdisk by optimising its geometrical parameters. Analogously to the FW-BIC formulation, this superposition can lead to the formation of a quasi-BIC state, whose Q-factor is bound only by the electromagnetic properties of the underlying multipole components. Quite remarkably, the field enhancement of quasi-BIC states can significantly boost non-linear photonic processes [20, 49, 50], and it can be employed to design single-particle nanolasers operating at cryogenic temperatures [51]. In these systems, the Q-factor could be enhanced by considering ensembles of dielectric nanoparticles. In the case of finite chains of resonators, for example, quasi-BICs can achieve a Q-factor as high as defect-based PhC microcavities [52]. These preliminary results could be further developed in integrated devices by introducing electrical injection and innovative material platforms such as hybrid perovskite [53–56]. Perovskites, in particular, could provide a high degree of tunability to the integrated lasers, opening to the realisation of stable and tunable integrated nanolasers relying on the amplification of radiationless states [57–59].

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